Ex2 : serie Oscillateur Libre

1) $Y_A \longrightarrow u(t) = u_B(t) + u_{R_2}(t)$

$Y_B \longrightarrow u_{R_1}(t) =$

à $t = 0s \Rightarrow i = 0 \Rightarrow u_{R_1}(0) = 0 \Rightarrow$

$\mathcal{C}_1 \longrightarrow u_{R_1}(t)$

$\mathcal{C}_2 \longrightarrow \cancel{u_B(t)}\ u(t)$

2) loi de Maille :

$u_{R_1} + u_{R_2} + u_b = E$

$(R_1+R_2+r)\, i + L\dfrac{di}{dt} = E$

En Régime Permanent :

$i = cst \Rightarrow \dfrac{di}{dt} = 0$

$\Rightarrow I_p = I_0 = \boxed{\dfrac{E}{R_1+R_2+r}}$

3) a) loi de Maille :

on pose $R_T = R_1 + R_2 + r$

$\left(R_T i + L\dfrac{di}{dt} = E\right) \times R_1$

$R_T u_{R_1} + L\dfrac{du_{R_1}}{dt} = R_1 E$

$\boxed{u_{R_1} = E - u}$

$R_T(E-u) + L\dfrac{d}{dt}(E-u) = R_1 E$

$R_T E - R_T u - L\dfrac{du}{dt} = R_1 E$

$R_T u + L\dfrac{du}{dt} = E(R_T - R_1)$

$R_T u + L\dfrac{du}{dt} = (R_2+r)\times E$

$$\boxed{u + \frac{L}{R_T}\frac{du}{dt} = \frac{R_2+r}{R_T}\cdot E}$$

equation differentielle

b) $u(t) = A + B e^{-t/\tau}$ est solᵒ

avec $\tau = \dfrac{L}{R_T}$

~~Remplaçant dans~~

à $t=0s$: $u(0) = u_b(0) + u_{R_2}(0) = E$

$A + B = E \Rightarrow \boxed{A = E - B}$

Remplaçant dans l'eq. diff :

$E - B + \cancel{Be^{-t/\tau}} + \tau\times\left(-\dfrac{1}{\tau}\right)\cancel{Be^{-t/\tau}} = \dfrac{R_2+r}{R_T}E$

$\Rightarrow \boxed{B = E - \dfrac{R_2+r}{R_T}E = \dfrac{R_1 E}{R_T} = R_1 I_0}$

$\boxed{A = E - B = E - \dfrac{R_1 E}{R_T} = \dfrac{(R_2+r)E}{R_T} = (R_2+r)I_0}$

$\Rightarrow \boxed{u(t) = (R_2+r)I_0 + R_1 I_0 e^{-t/\tau}}$

c) $u_{R_1} = E - u$

$u_{R_1} = E - \dfrac{R_2+r}{R_T}E - \dfrac{R_1 E}{R_T}e^{-t/\tau}$

$u_{R_1}(t) = \dfrac{R_1 E}{R_T} - \dfrac{R_1 E}{R_T}e^{-t/\tau}$

$u_{R_1}(t) = \dfrac{R_1 E}{R_T}\left(1 - e^{-t/\tau}\right)$

$\boxed{u_{R_1}(t) = R_1 I_0\left(1 - e^{-t/\tau}\right)}$

4) $U_{Bp} = 0{,}2\,V$ et $I_0 = I_p = 40\,mA$

- $U_{Bp} = r I_0 \Rightarrow r = \dfrac{U_{Bp}}{I_0} = \dfrac{0{,}2}{40.10^{-3}}$

$\boxed{r = 5\Omega}$

- D'après la Courbe $\mathcal{C}_2 \Rightarrow U_p = 2V$

$\Rightarrow U_{R_2p} = U_p - U_{Bp} = 2 - 0{,}2 = 1{,}8V$

$\Rightarrow R_2 I_p = 1{,}8 \Rightarrow R_2 = \dfrac{1{,}8}{40.10^{-3}}$

$\boxed{R_2 = 45\Omega}$

5) à $t = t_1 \Rightarrow u(t) = u_{R_1}(t)$

a) $u + u_{R_1} = E \Rightarrow 2u_{R_1} = E$

$u_{R_1}(t_1) = \frac{E}{2} \Rightarrow$

$\frac{R_1 E}{R_T}\left(1 - e^{-t_1/\tau}\right) = \frac{E}{2}$

$1 - e^{-t_1/\tau} = \frac{R_T}{2R_1}$

$e^{-t_1/\tau} = 1 - \frac{R_T}{2R_1} = \dots$

$= \frac{2R_1 - R_1 - R_2 - r}{2R_1}$

$= \frac{R_1 - R_2 - r}{2R_1}$

$-\frac{t_1}{\tau} = \ln\left(\frac{R_1 - R_2 - r}{2R_1}\right)$

$\boxed{t_1 = -\tau \ln\left(\frac{R_1 - R_2 - r}{2R_1}\right)}$

$U_{R_1}(t_1) = \frac{R_1 E}{R_T}\left(1 - e^{\ln\left(\frac{R_1 - R_2 - r}{2R_1}\right)}\right)$

$U_{R_1}(t_1) = \frac{R_1 E}{R_T}\left[1 - \frac{R_1 - R_2 - r}{2R_1}\right]$

$= \frac{R_1 E}{R_T}\left[\frac{2R_1 - R_1 + R_2 + r}{2R_1}\right]$

$= \frac{\cancel{R_1} E}{\cancel{R_T}} \times \frac{\cancel{R_T}}{2\cancel{R_1}} = \frac{E}{2}$

$\Rightarrow \boxed{E = 2U_{R_1}(t_1) = 2 \times 3 = 6V}$

c) $U_{R_1p} = E - U_p = 6 - 2 = 4V$

$R_1 I_0 = U_{R_1p} \Rightarrow R_1 = \frac{U_{R_1p}}{I_0}$

$\boxed{R_1 = \frac{4}{40 \cdot 10^{-3}} = 100\Omega}$

τ est l'abscisse du pt d'intersection de la tangente à l'origine à la courbe $u_{R_1}(t)$ avec la droite $u_{R_1p} = 4V \Rightarrow$

$\tau = 2ms = 2 \cdot 10^{-3} s$

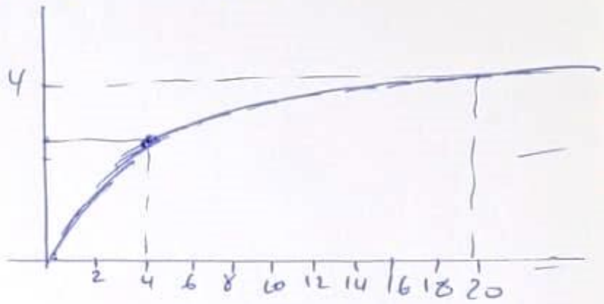
$L = \tau \times R_T = 2 \cdot 10^{-3}[100 + 45 + 5]$

$\boxed{L = 0,3H}$

6) $L' = 2L \Rightarrow \tau' = 2\tau = 4ms$

$\Rightarrow 5\tau' = 5 \times 4 = 20ms$

pour $t = \tau' \Rightarrow U_{R_1}(\tau') = 0,63 \times 4 = 2,5V$



B) 1) loi des mailles :

$(R_2 + r)i + u_C + L\frac{di}{dt} = 0$

$\boxed{u_C + (R_2 + r)C\frac{du_C}{dt} + LC\frac{d^2u_C}{dt^2} = 0}$

2) Libre —

Amorti —

3) a) $E_T = E_C + E_L = \frac{1}{2}Cu_C^2 + \frac{1}{2}Li^2$

$\frac{dE_T}{dt} = -(R_2 + r)i^2 < 0 \Rightarrow E_T \searrow$

4) $T_0^2 = 4\pi^2 LC \Rightarrow C = \frac{T_0^2}{4\pi^2 L}$

$C = \frac{(8 \cdot 10^{-3})^2}{4\pi^2 \times 0,3} = 5,4 \times 10^{-6} F$

5) $W_J = E_T(0) - E_T(t_1)$

• $E_T(0) = \frac{1}{2}CU^2 = \frac{1}{2} \times 5,4 \times 10^{-6} \times 12^2 = 3,88 \cdot 10^{-4} J$

• $E_T(t_1) = \frac{1}{2}Cu_C^2(t_1) + \frac{1}{2}Li^2$

$i(t_1) = C\frac{du_C}{dt}(t_1) = C \times$ pente